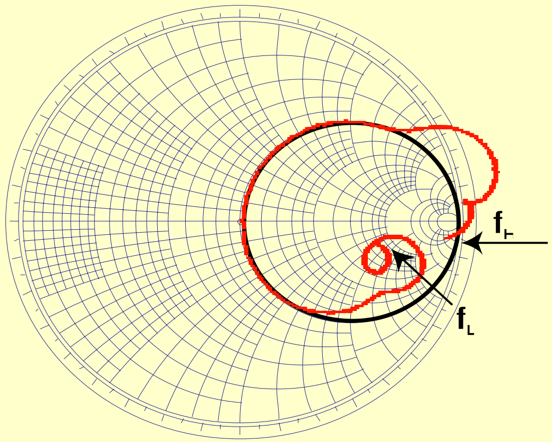

Sensitivity of the rf-set

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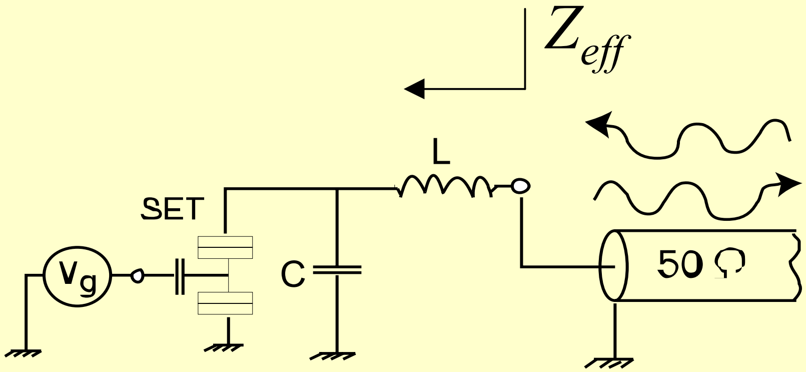
Outline

- short introduction to rf-SET
- Signal-to-noise ratio and SET charge sensitivity
- Noise power waves
- experimental verification

rf-SET: *R. Schoelkopf et. al , Science (1999)*



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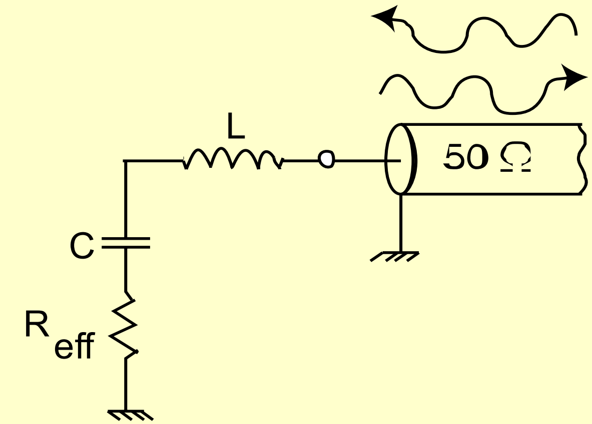
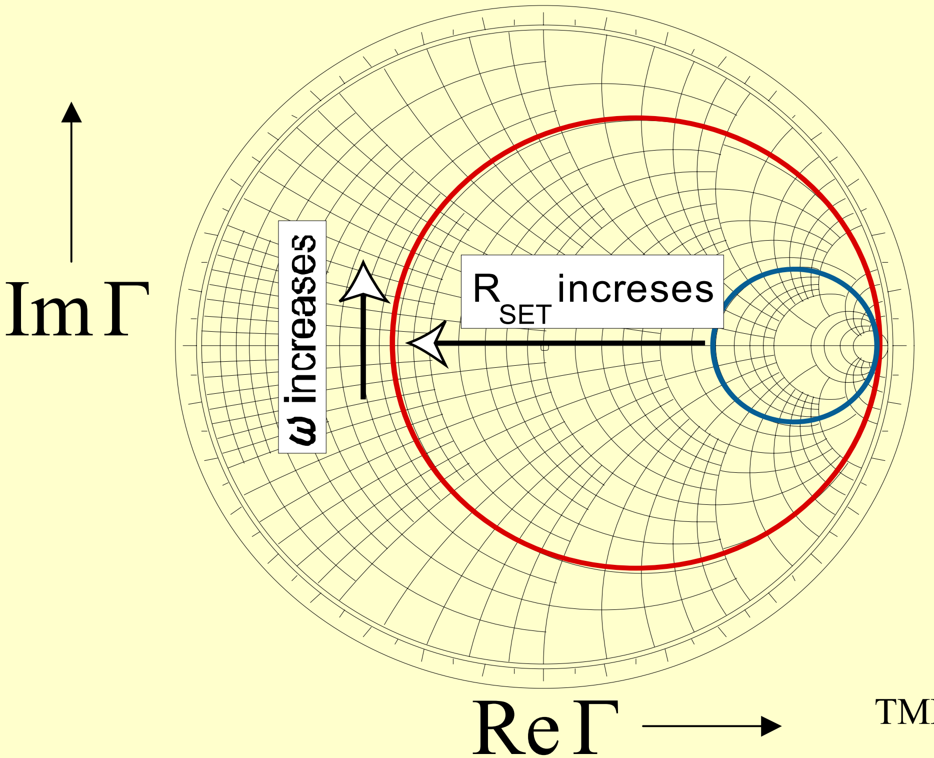


Reflected wave complex amplitude Γ



generally: $$Z_{eff} = R_{eff} + j\frac{2Z_T\Delta\omega}{\omega_0}$$

at resonance: $$R_{eff} = \frac{L}{CR_{SET}} \equiv Z_T^2 / R_{SET}$$

$$\Gamma = \frac{Z_{eff} - 50}{Z_{eff} + 50}$$

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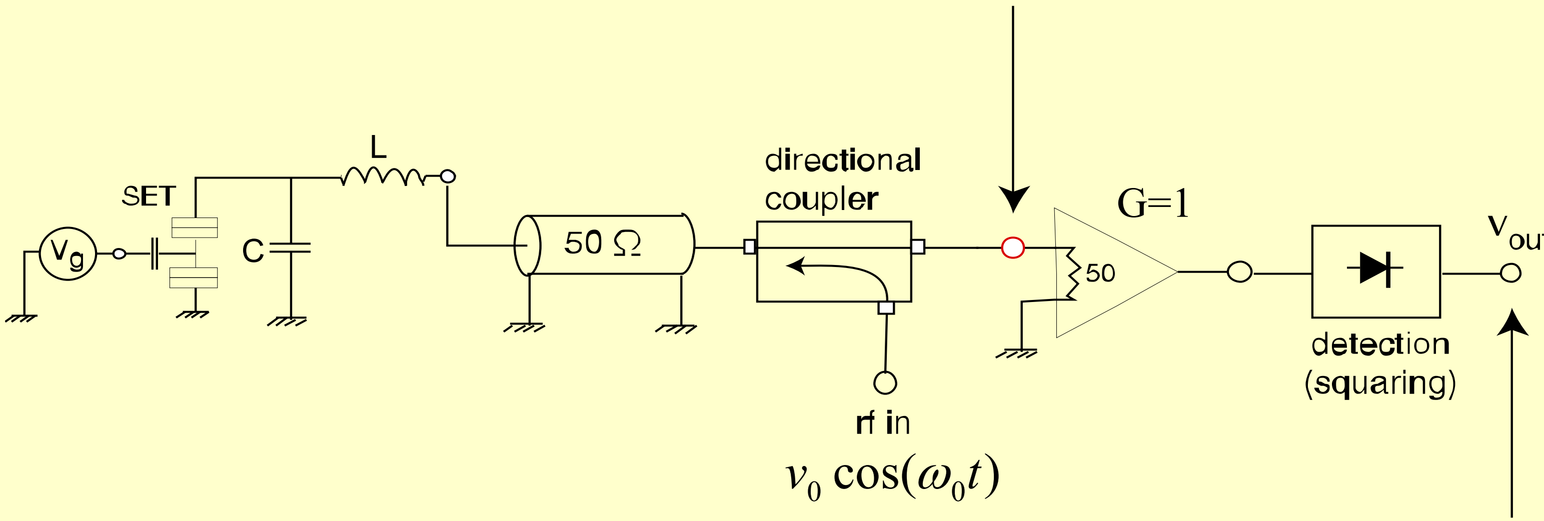
Signal-to-noise ratio

$$v_0[\Gamma_0 + \Delta\Gamma\cos(\omega_m t)]\cos(\omega_0 t) + n(t)$$



$$v_0\cos(\omega_0 t)$$

$$v_0\frac{\Delta\Gamma}{2}\cos(\omega_m t) + n(t)\cos(\omega_0 t)$$

input noise *n(t)* equivalent noise temp. T_0
such that $S_V = k_B T Z_0$.

output noise $n(t)\cos(\omega_0 t)$ =>
$S_{V,output} = k_B T Z_0/2$

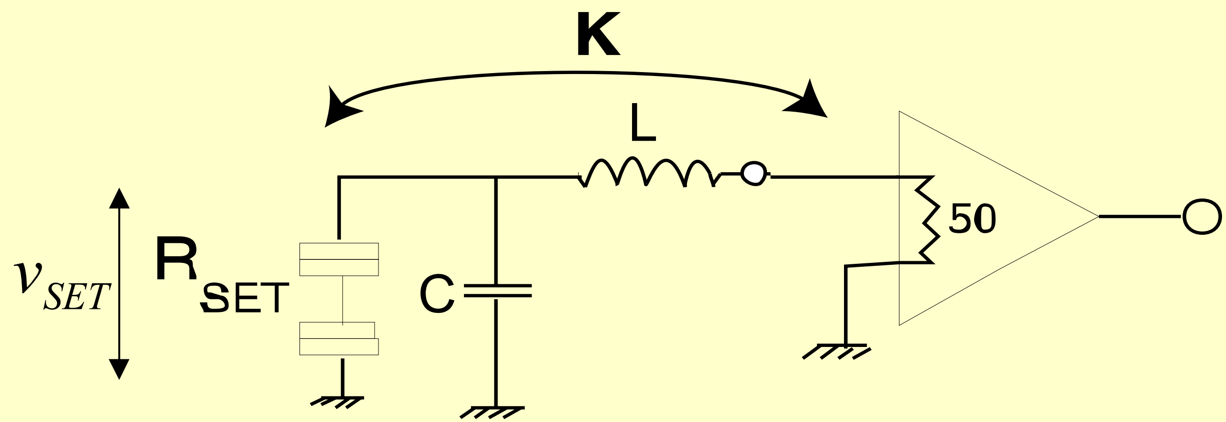
$$\frac{S}{N} = \frac{v_0\Delta\Gamma}{\sqrt{4k_B T_0 Z_0}}$$

$$\frac{S}{N} = \frac{v_0 \Delta\Gamma}{\sqrt{4k_B T_0 Z_0}}$$

K: coupling of available power between impedances.

K=1 is conjugate matching

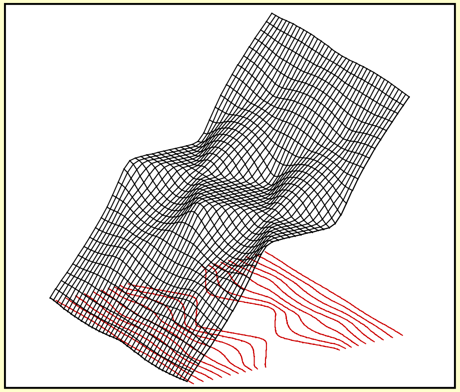
$$\Delta\Gamma = \frac{\partial |\Gamma|}{\partial q} \delta q$$



$$\frac{v_0^2}{2Z_0} \mathbf{K} = \frac{v_{SET}^2}{2R_{SET}}$$

$$\delta q_{RMS} = \frac{\sqrt{2k_B T_0 R_{SET} K}}{v_{SET} \dfrac{\partial |\Gamma|}{\partial q}}$$

Numerical estimation of charge and energy sensitivity.

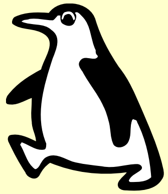


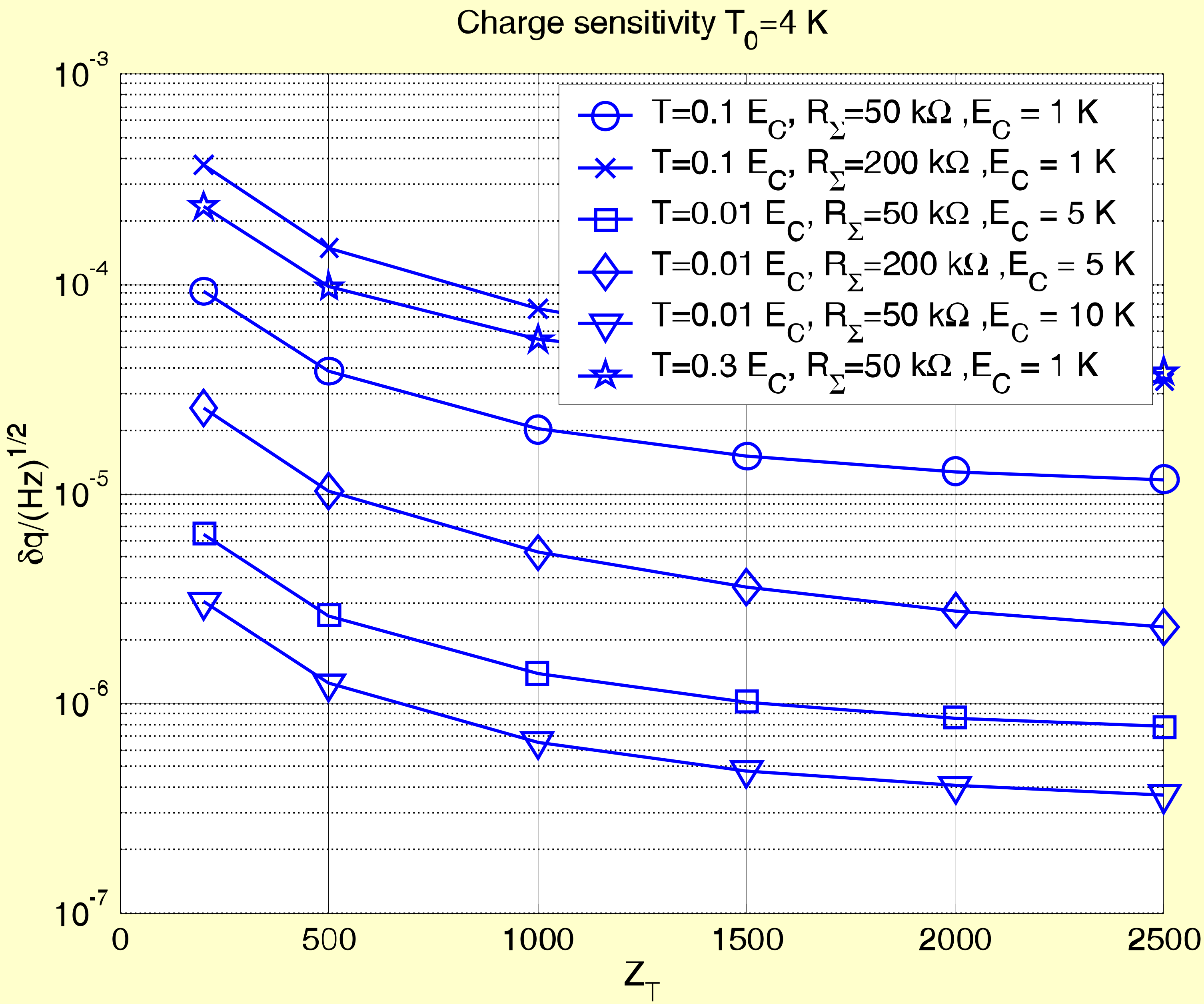
1) calculate $I(V_{BIAS,} V_{GATE})$ curves using orthodox theory. Parameters E_C, T, R_{sigma} .

2) calculate $R(V_{AC}, V_{GATE})$ using $$R \approx \sqrt{\frac{<U^2>}{<I^2>}}$$

3) minimize $$\delta q_{RMS} = \frac{\sqrt{2k_B T_0 R_{SET} K}}{v_{SET} \frac{\partial |\Gamma|}{\partial q}}$$ w.r.t. V_{AC} and V_{GATE}

4) Results charge sensitivity limited by amplifier





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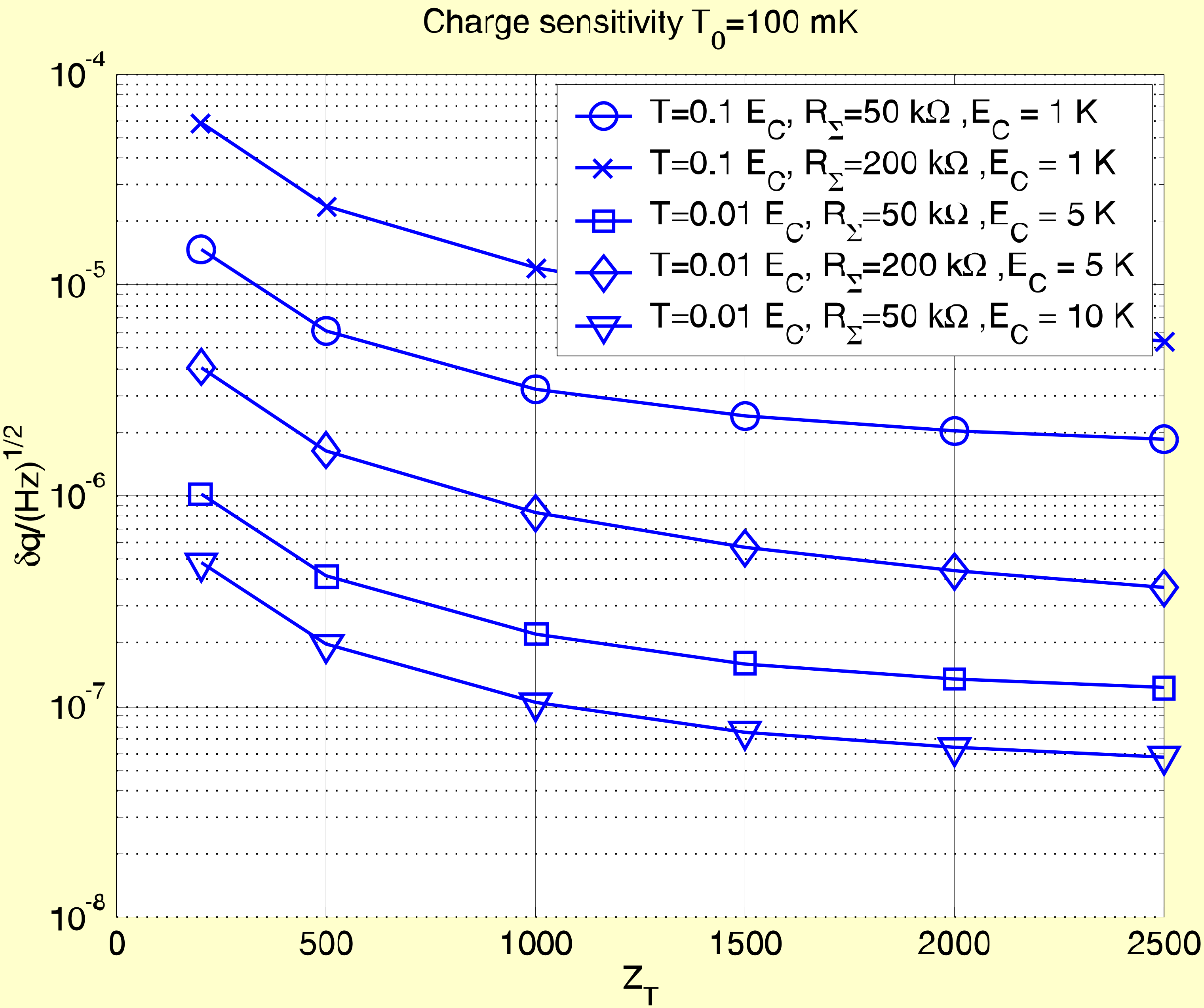

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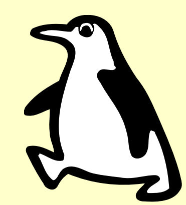
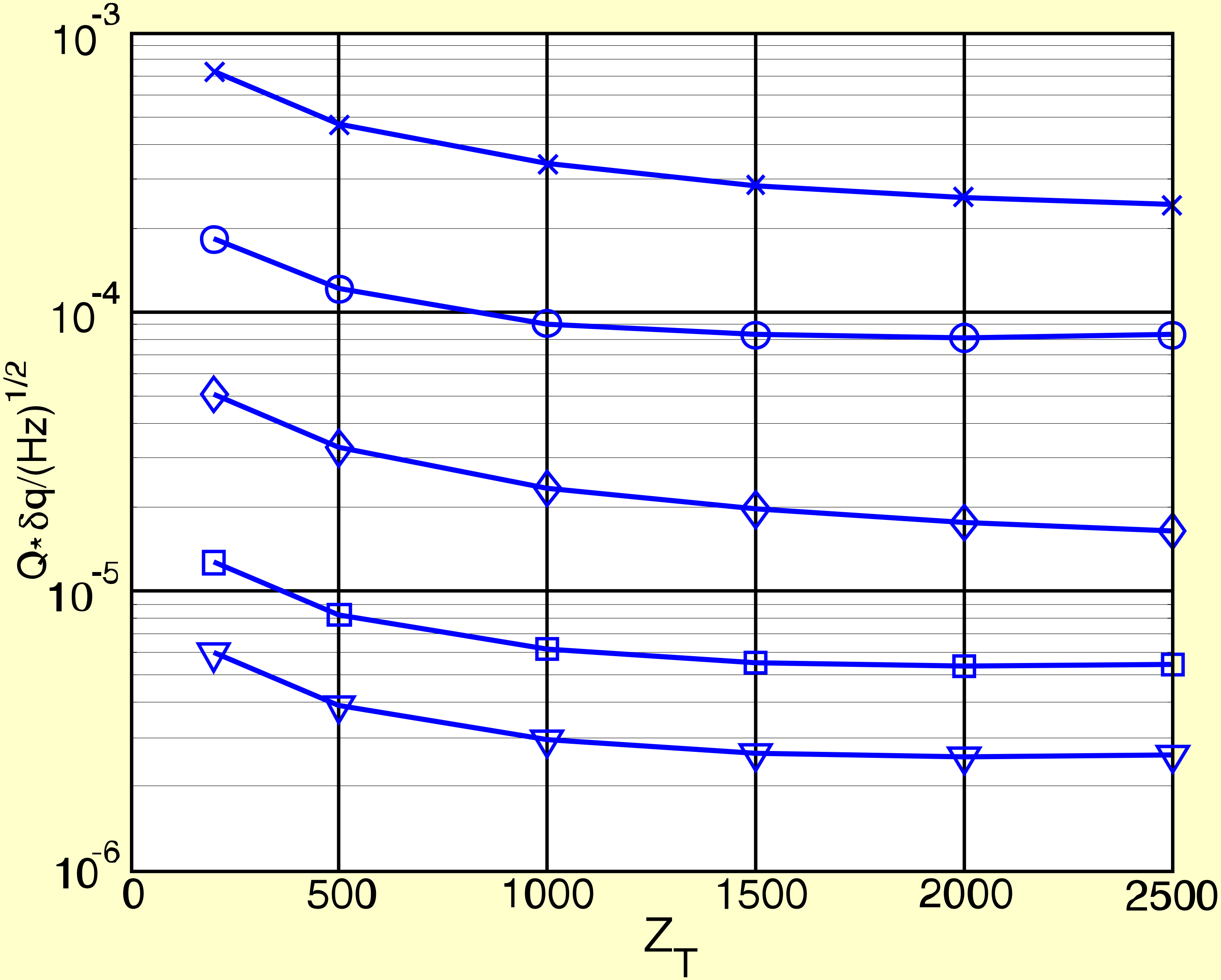


Inverse gain-bandwidth product (lower is better)



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Rule of thumb formula for charge sensitivity

$$\delta q \approx 1.46 \cdot 10^{-6} Z_T^{-0.91} t^{0.59} T_{EC}^{-1.01} R_\Sigma^{0.91} T_0^{0.5} [e/\sqrt{Hz}]$$

$$t = T / T_{EC}$$

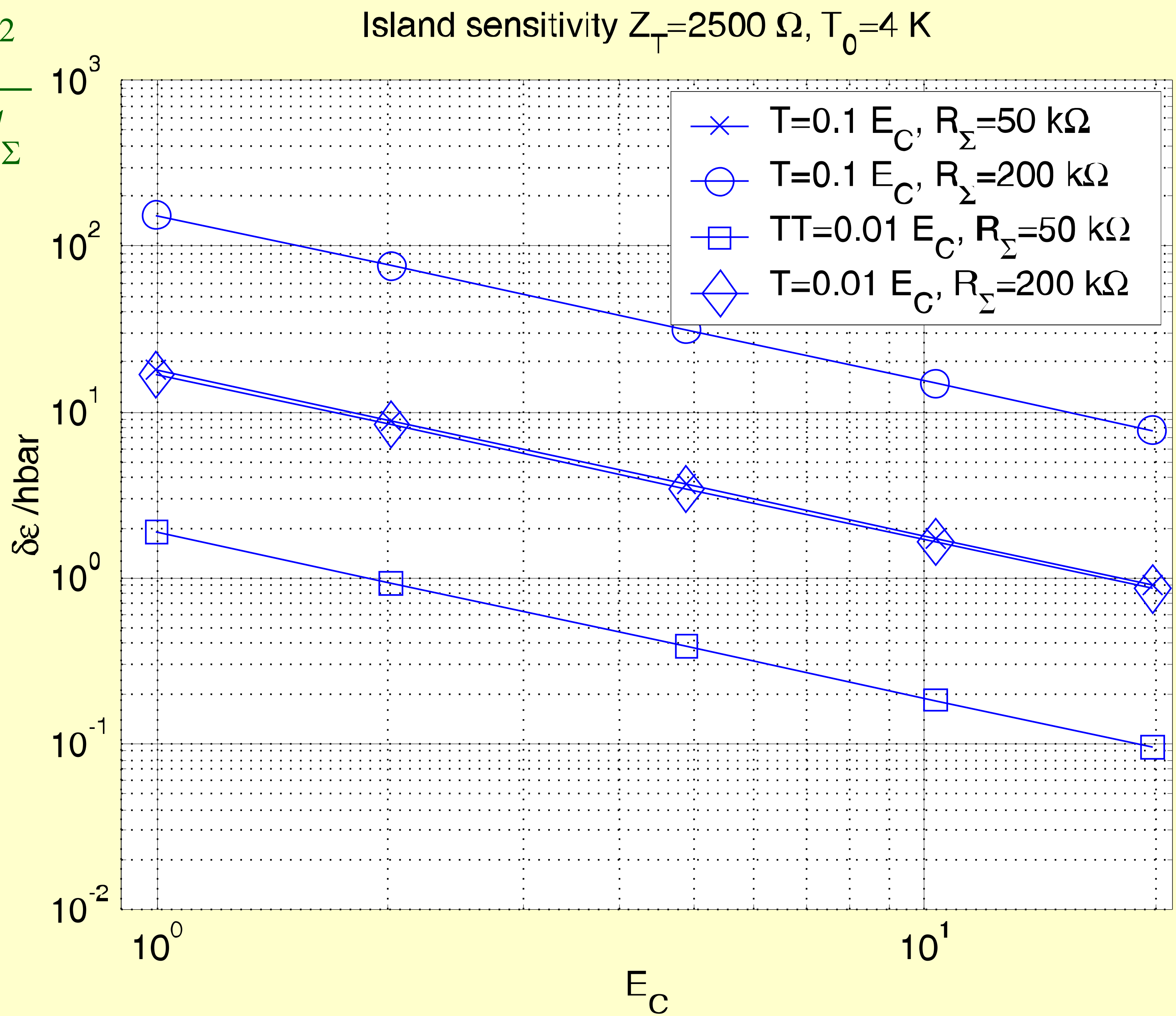
$$Z_T = \sqrt{L/C}$$

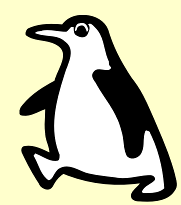
amplifier noise

SET total resistance

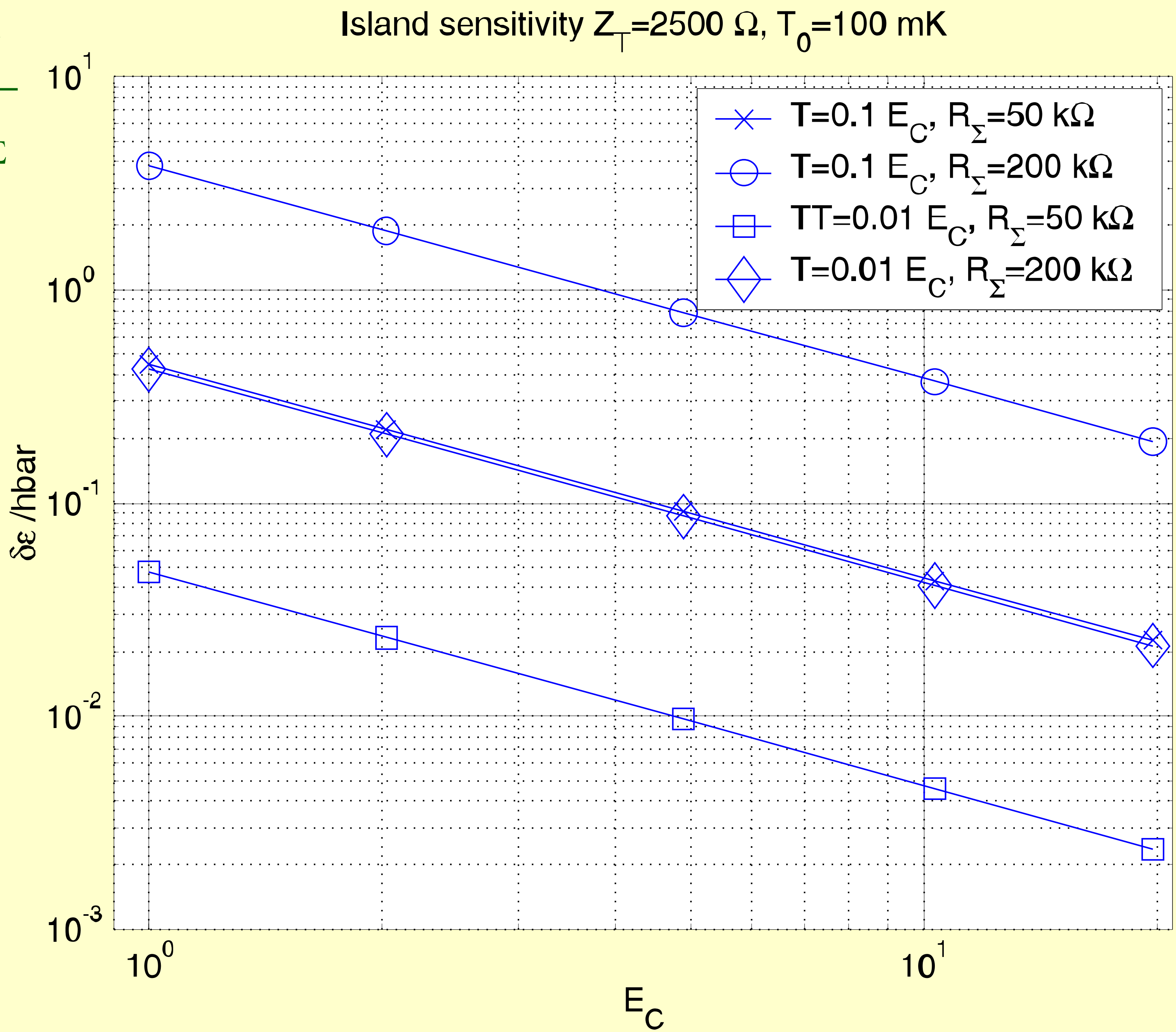
charging energy in Kelvins

$$\delta\varepsilon = \frac{\delta q^2}{2C_\Sigma}$$





$$\delta\varepsilon = \frac{\delta q^2}{2C_\Sigma}$$



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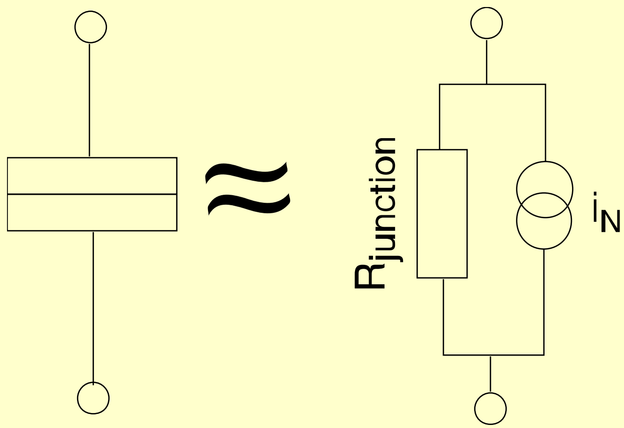
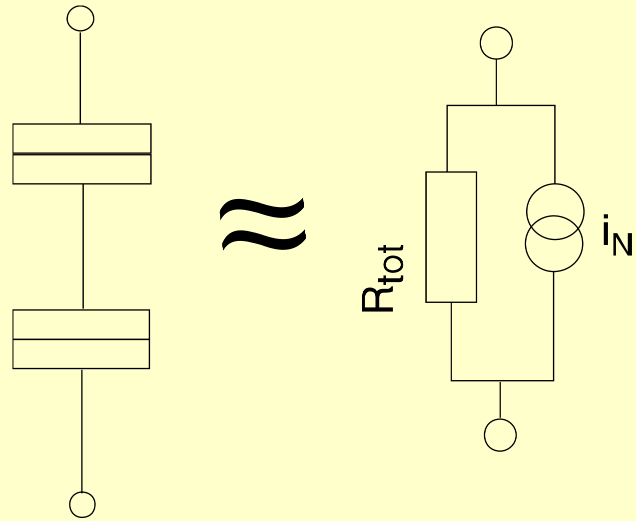
SET shot noise (as calibration standard)





$$S_I = \frac{2eV}{R_{junction}} \coth \frac{eV}{2k_B T}$$

$$S_I = \frac{eV}{R_{tot}} \coth \frac{eV}{4k_B T}$$

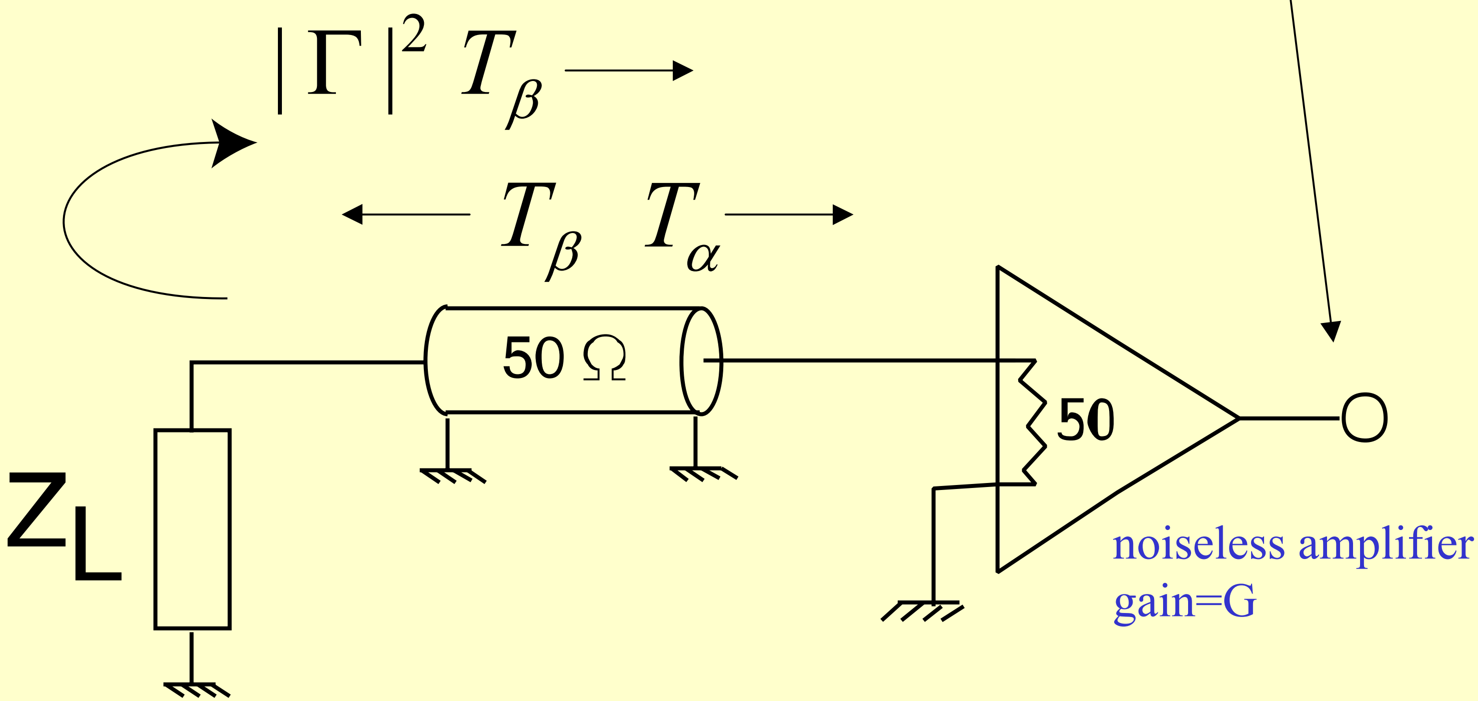
assuming junctions uncorrelated

available power: $S_I R / 4$

NOISE POWER WAVES (=amplifier noise)

(correlation term)

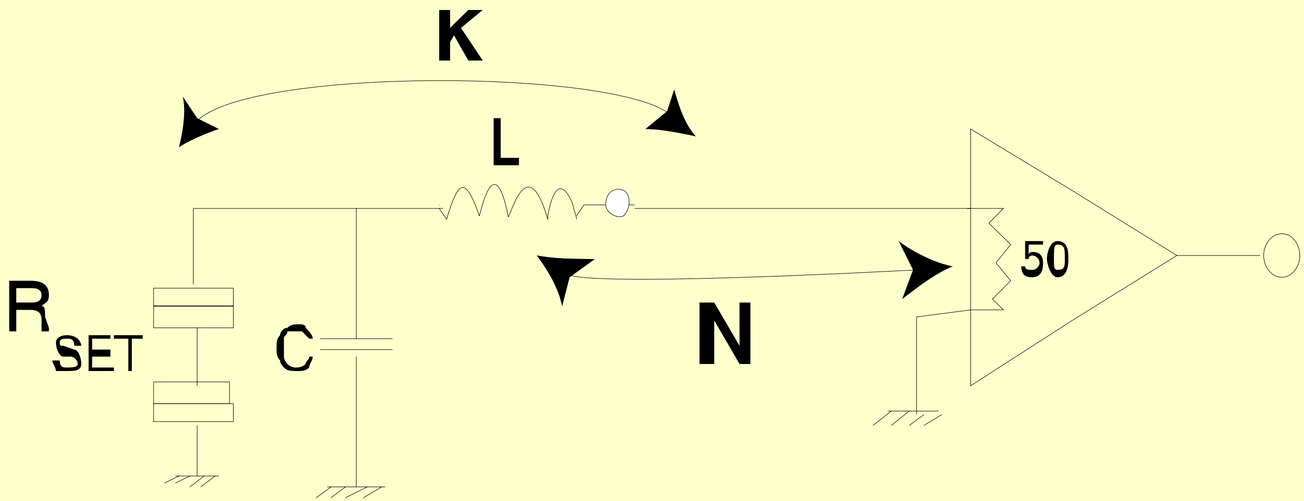
$$G \cdot \left\{ \; T_\alpha + |\Gamma|^2 \; T_\beta - 2\,|\Gamma|\, T_\gamma \cos(\phi_\gamma + \phi_L) \right\}$$



Coupling of available power between objects

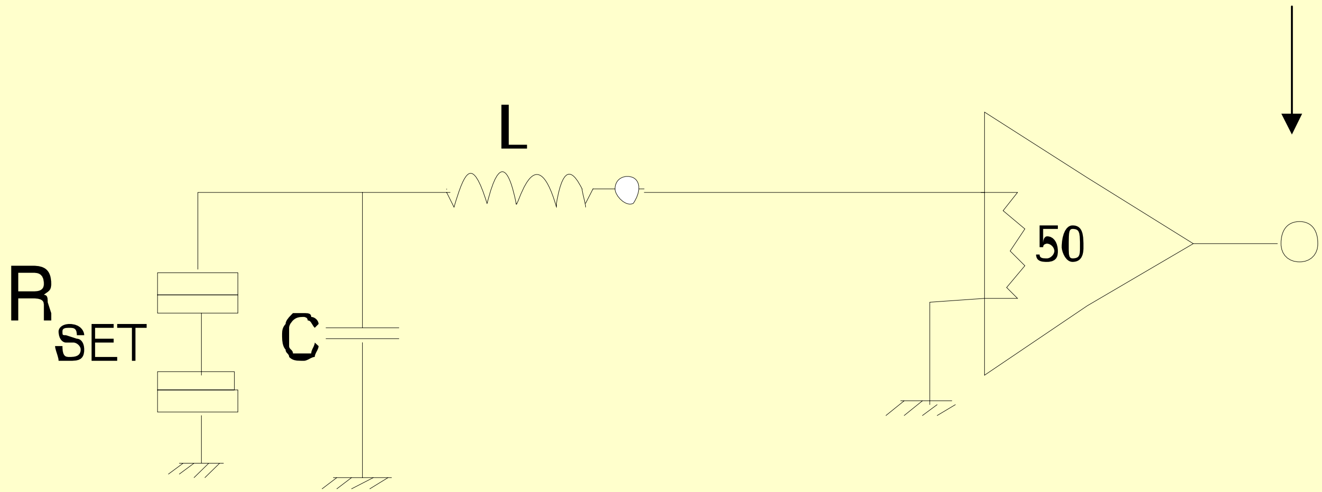
K: between SET and amplifier input

N: between LC tank circuit and amplifier input



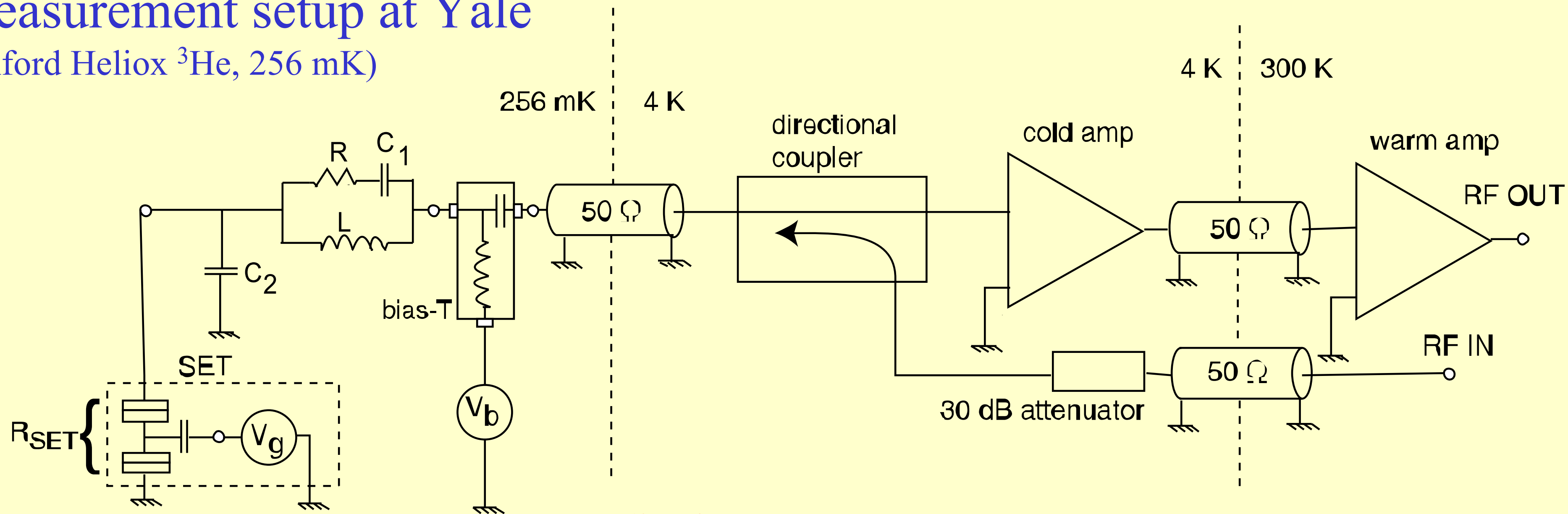
Measured total noise

$$G \cdot k_B \left\{ T_\alpha + |\Gamma|^2\, T_\beta - 2|\Gamma|\, T_\gamma \cos(\phi_\gamma + \phi_L) + \mathbf{N} T_N + \mathbf{K} \frac{eV}{4k_B} \coth \frac{eV}{4k_B T} \right\}$$



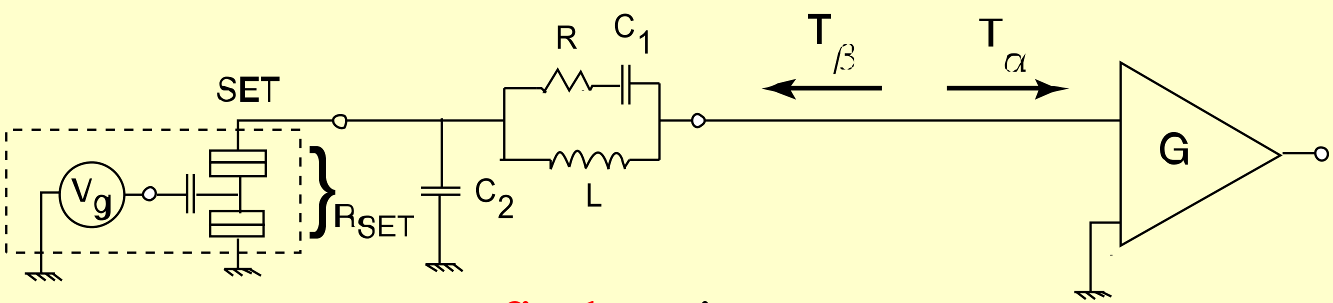
Measurement setup at Yale

(Oxford Heliox ^{3}He, 256 mK)

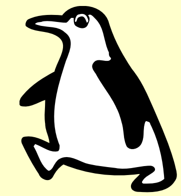


find: C_1, C_2, L, R

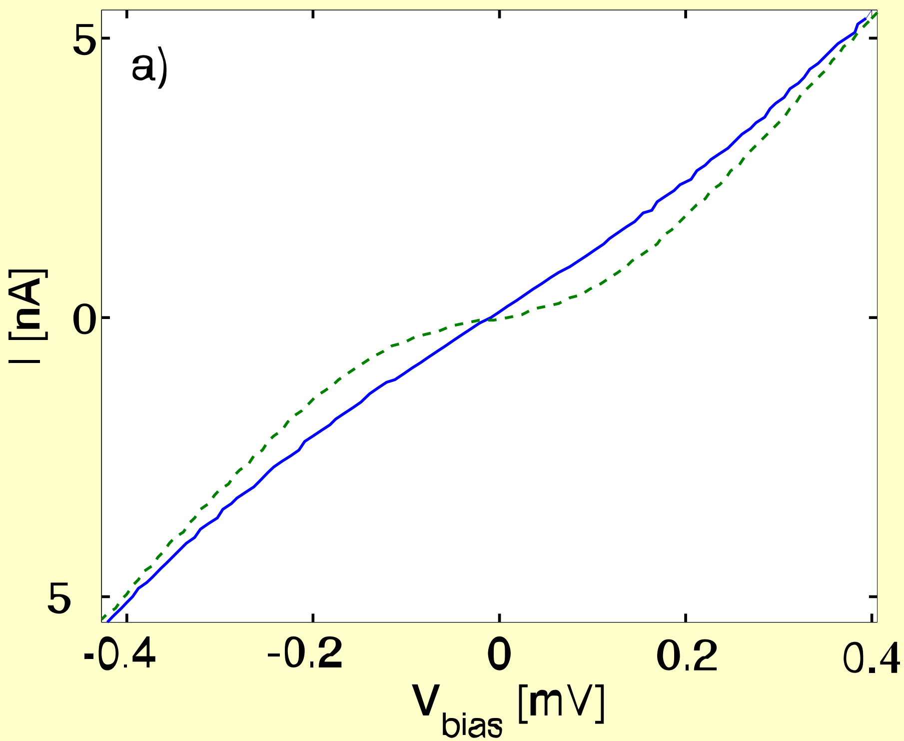
Equivalent noise setup



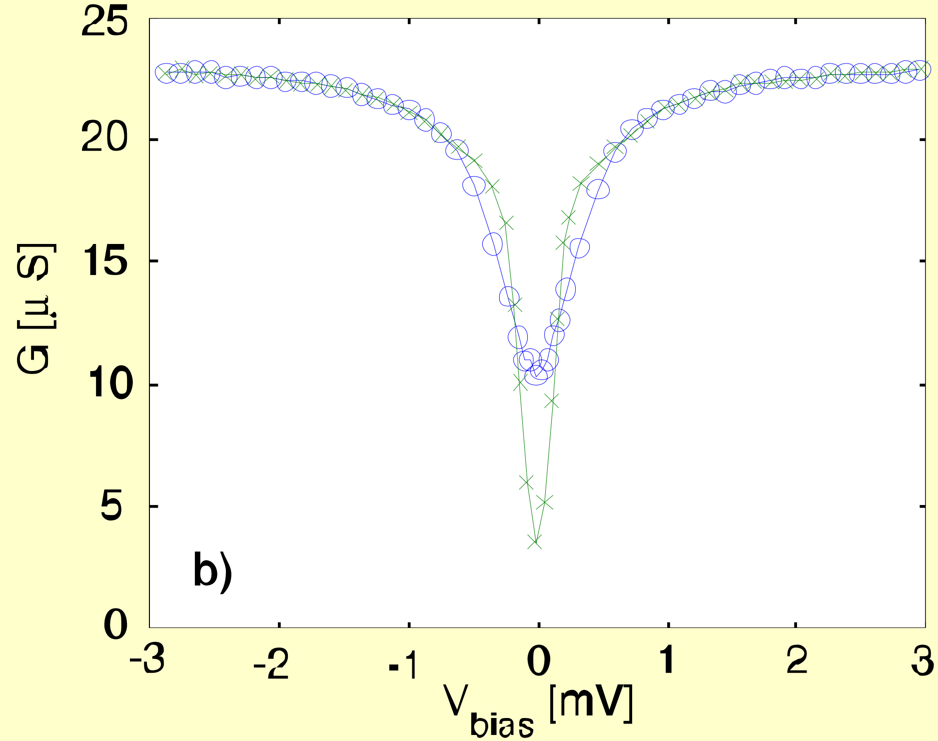
find: noise parameters



DC IV Curves



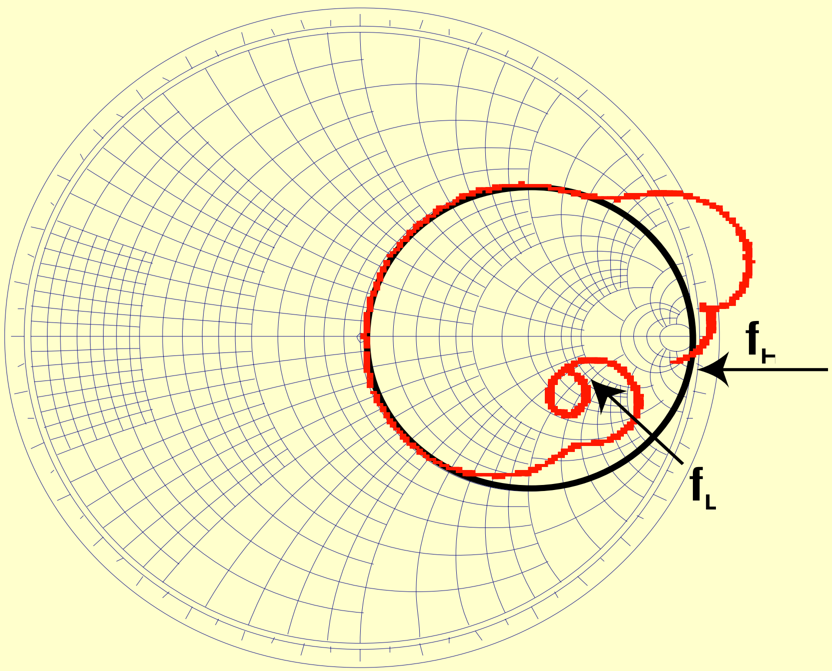
Conductance

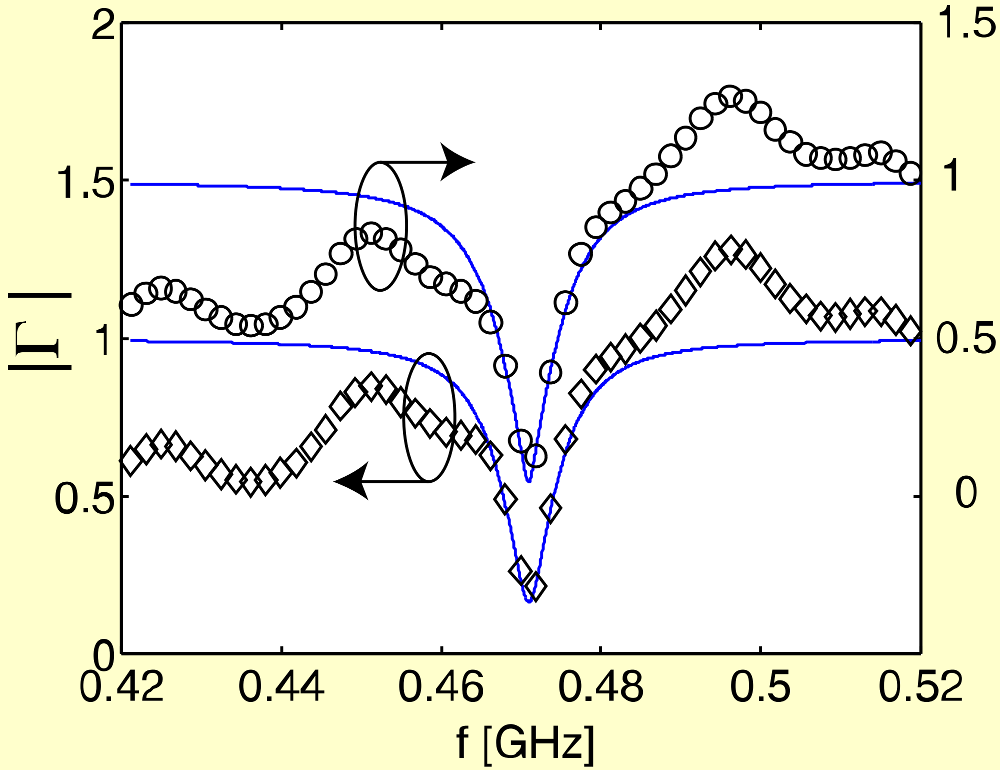


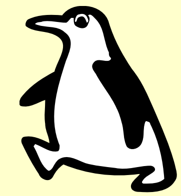
sample: Al SET

=> E_C=1.13 K

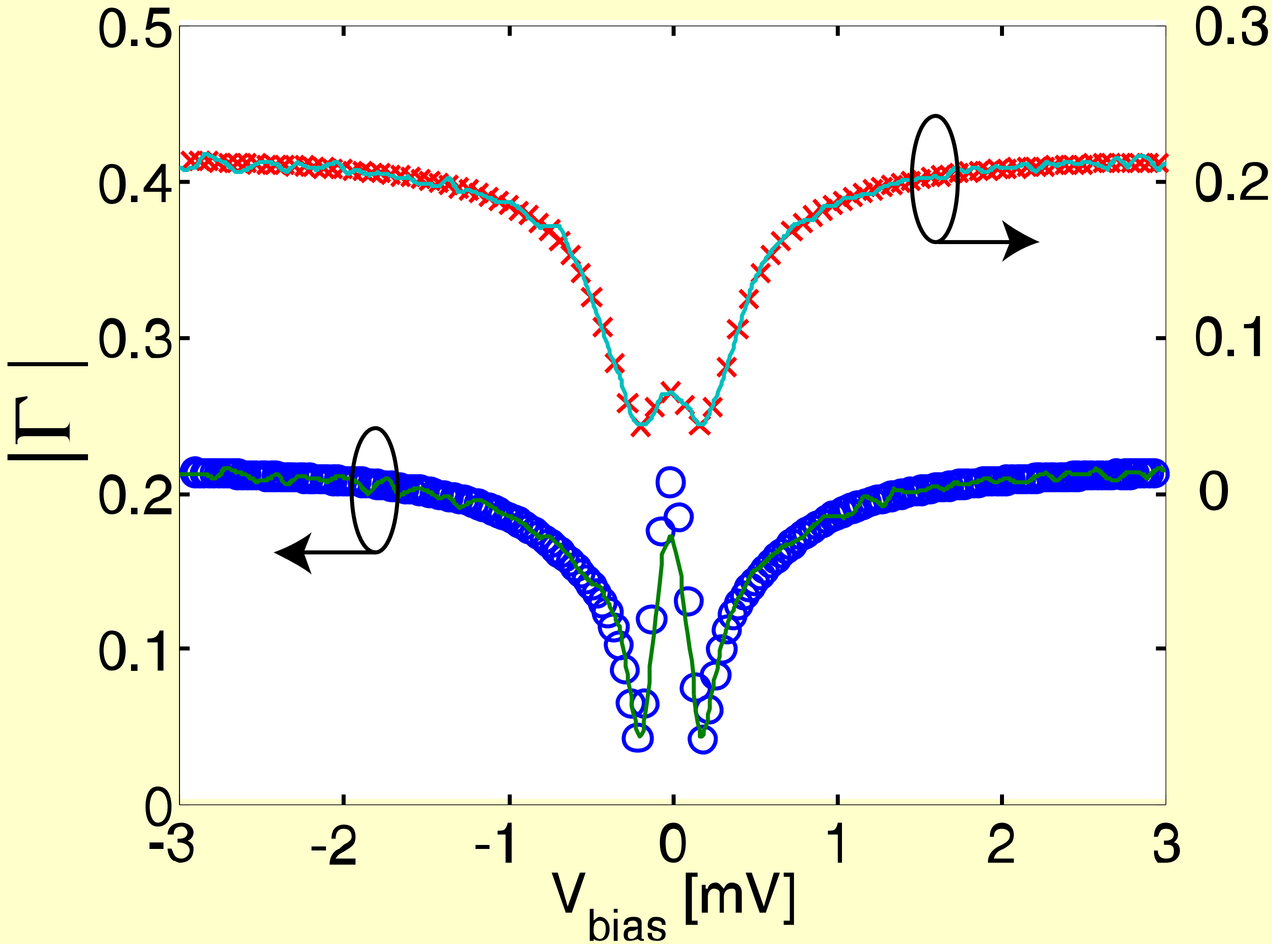
Frequency response:

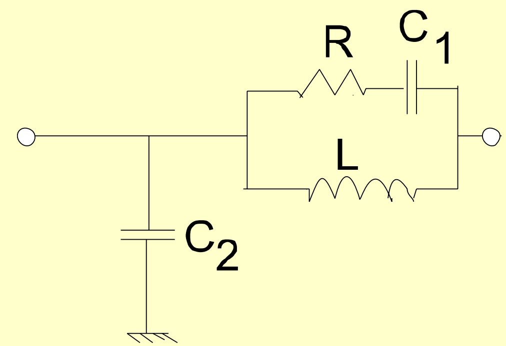






gamma vs. V_{bias}





=>

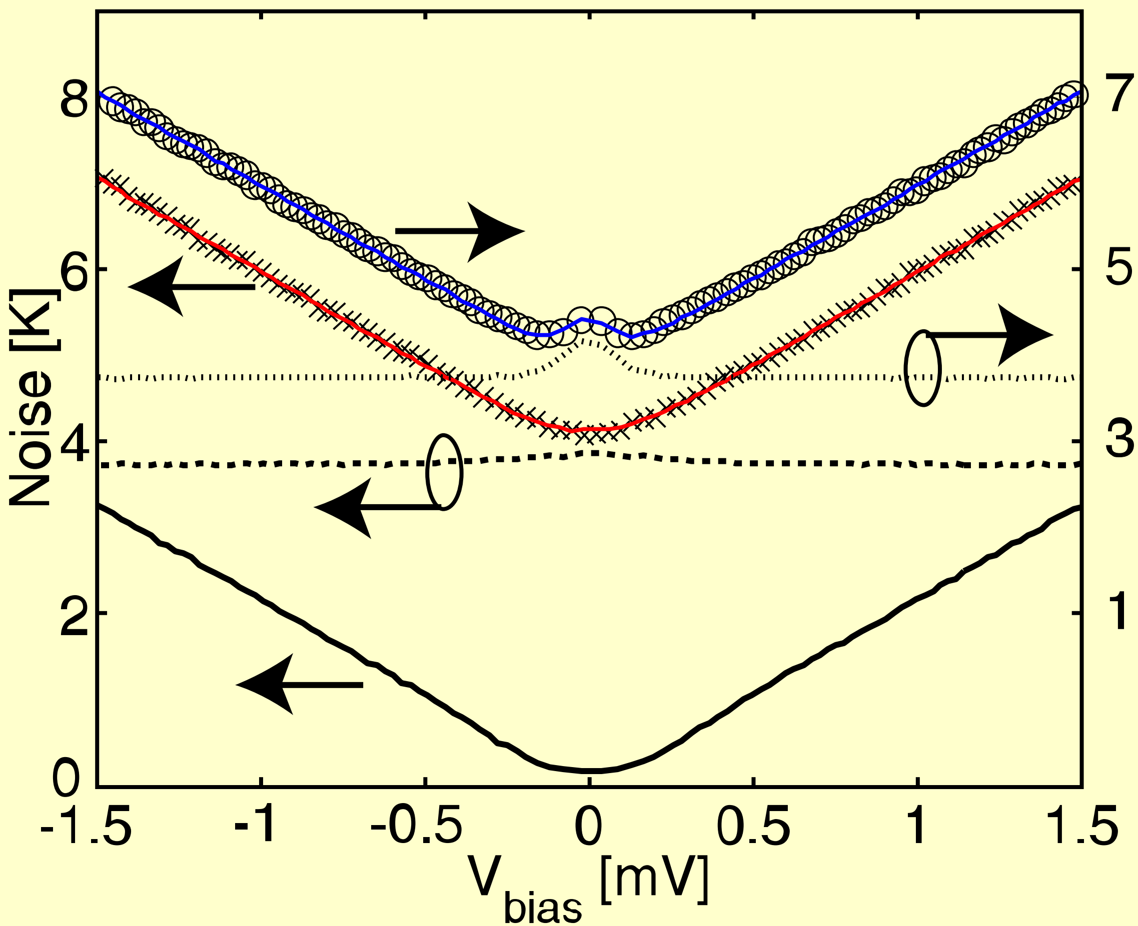
C_1= 0.239 pF

C_2 = 0.208 pF

L = 256 nH

R =12.0 Ohm

using C_1, C_2, L, R to fit noise parameters:



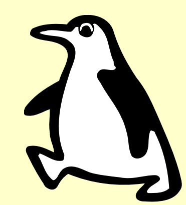
$$T_\alpha = 3.78$$

$$T_\beta = 1.22$$

$$T_\gamma = 0.35$$

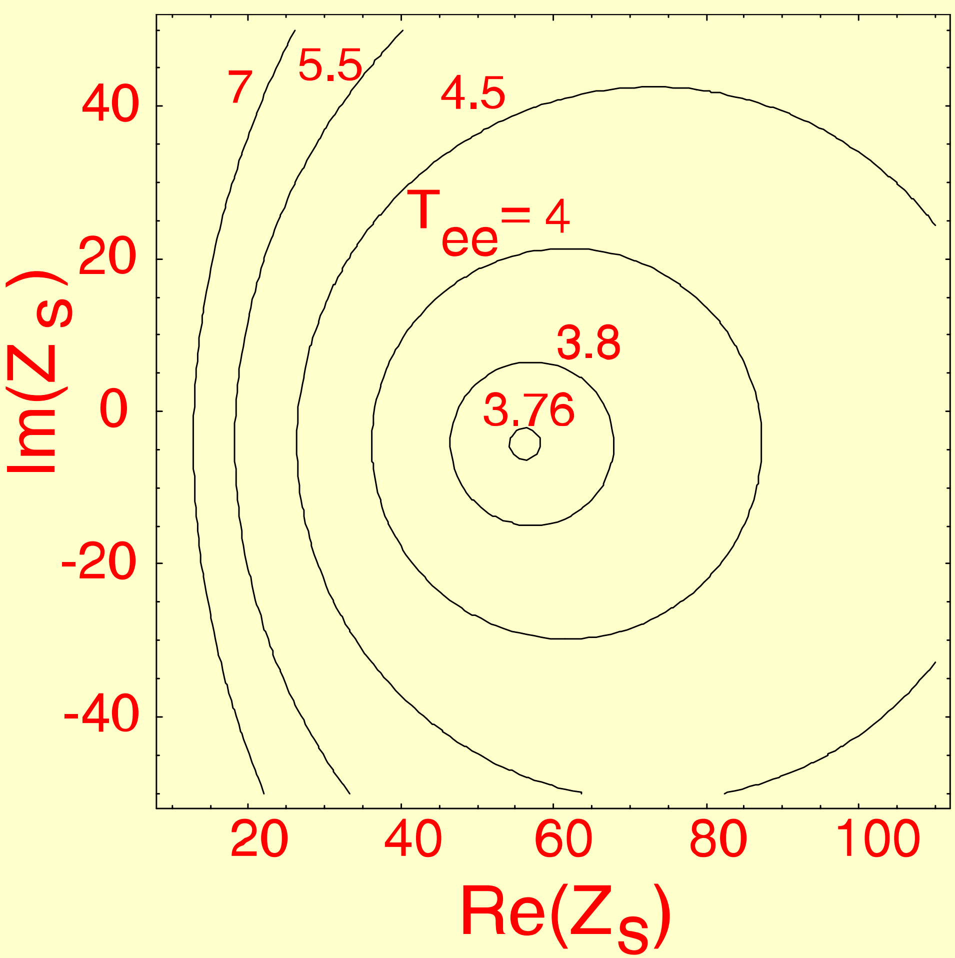
$$\phi_\gamma = 0.56$$

$$\Rightarrow T_0 \approx 4K$$

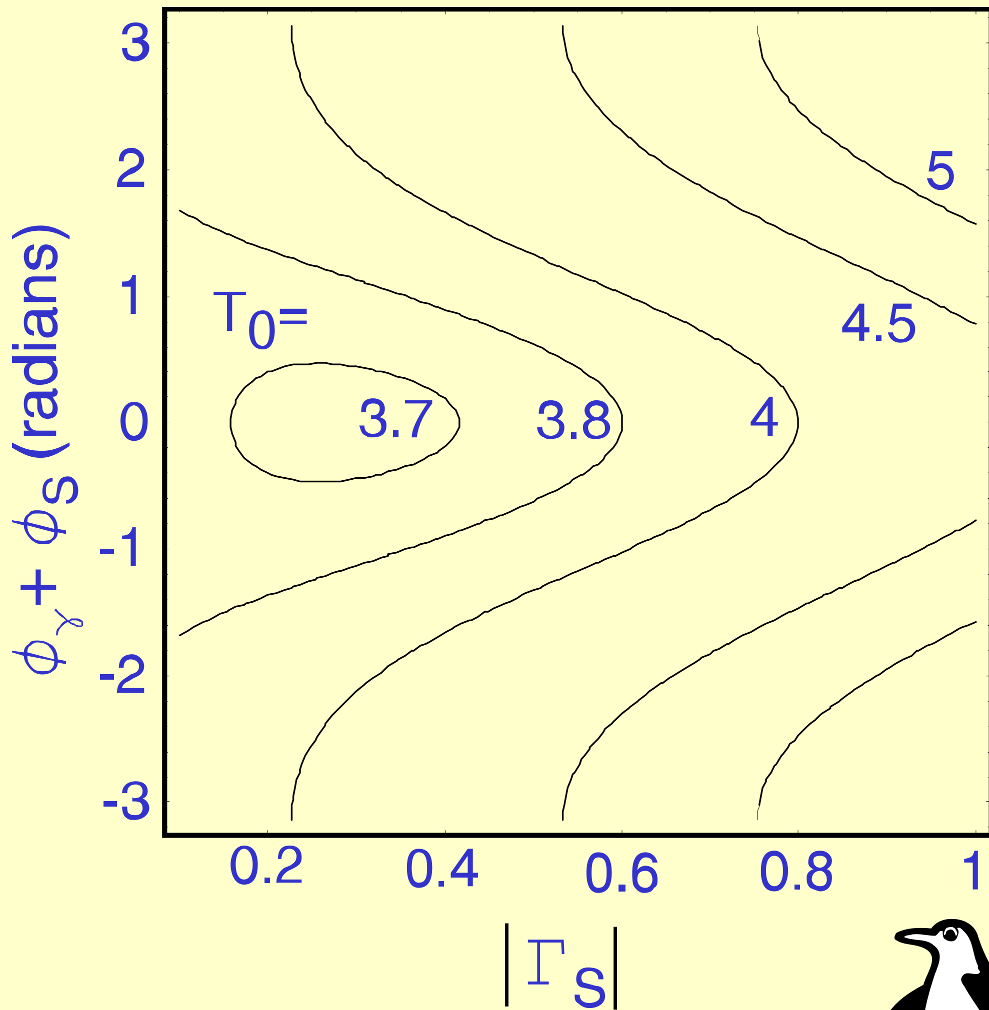


Amplifier noise contours

effective extended noise temperature



amplifier input noise temperature



Comparison of $\delta q_{RMS} = \dfrac{\sqrt{2k_B T_0 R_{SET} K}}{v_{SET} \dfrac{\partial |\Gamma|}{\partial q}}$ to measured charge sensitivity.

extracted parameters: R_{SET}, K, T_0, $\Gamma(C_1, C_2, L, R, q)$

$$\Rightarrow \delta q_{calculated} = 3.5 \cdot 10^{-5} e / \sqrt{Hz}$$

$$\Rightarrow \delta q_{measured} = 3.8 \cdot 10^{-5} e / \sqrt{Hz}$$

Conclusion

- full model explaining rf-SET system
- estimation of charge sensitivity
- => lower noise amplifiers needed in order to reach shot-noise limit

$$\delta q_{RMS} = \frac{\sqrt{2k_B T_0 R_{SET} K}}{v_{SET} \dfrac{\partial |\Gamma|}{\partial q}}$$

$$\delta q \approx 1.46 \cdot 10^{-6} Z_T^{-0.91} t^{0.59} T_{EC}^{-1.01} R_\Sigma^{0.91} T_0^{0.5} [e/\sqrt{Hz}]$$